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SOME RELATED PROBLEMS OF FILTRATION AND HEAT CONDUCTION IN POROUS BODIES

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An examination is made of a dynamically similar boundary-value problem describing the uniform motion of a liquid or gas initiated by intensive heat flow to a porous body.

1. We will examine the motion of a uniform liquid in a porous body under nonisothermal conditions. We will assume that the time required to establish local thermal equilibrium is short and that we can use a one-temperature model. We then have the system of equations (see [1], for example):

$$-\frac{k\rho}{\mu}\nabla p = \Phi(J)\,\mathbf{J}/J, \tag{1}$$

$$\frac{\partial (m\rho)}{\partial t} + \operatorname{div}\mathbf{J} = 0, \tag{2}$$

$$\frac{\partial (i\rho m + CT)}{\partial t} + \operatorname{div}(i\mathbf{J}) + \operatorname{div}\mathbf{q} = 0, \tag{3}$$

$$\mathbf{q} = -\lambda\nabla T,\quad i = i(p,\ T),\quad \rho = \rho(p,\ T),\quad \lambda = \lambda(p,\ T),\quad \mu = \mu(p,\ T). \tag{4}$$

The function $\Phi(J)$ describes the filtration law. With the chosen form, system (1) covers a variety of cases of nonisothermal motion of liquids and gases in a porous medium with both a linear and a nonlinear filtration law. It was used in [1] to study temperature changes connected with the Joule–Thompson effect in the nonsteady flow of gas to wells. Below we examine what is in a sense the opposite problem: the motion of a liquid or gas initiated by intensive heat flow to a porous body.

2. Let a half-space $x \geq 0$ at the initial moment of time be filled with a moving gas with a constant pressure p_0 and temperature T_0, and let certain new values of pressure and temperature p_1 and T_1 be established at the boundary beginning with the moment $t = 0$. The resulting unidimensional motion satisfies the conditions

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$$p(x,\ 0)=p_0,\quad T(x,\ 0)=T_0,\quad p(0,\ t)=p_1,\quad T(0,\ t)=T_1. \tag{5}$$

If the Darcy law $[\Phi(j)\equiv j]$ is valid, then it is easy to see that the motion is dynamically similar and can be written as

$$p(x,\ t)=p(\xi),\quad T(x,\ t)=T(\xi),\quad \xi=x/a\sqrt{t}, \tag{6}$$

where $p(\xi)$ and $T(\xi)$ satisfy the system of equations

$$\begin{gathered}\frac{d}{d\xi}\left(\frac{k\rho}{\mu m}\frac{dp}{d\xi}\right)+\frac{a^2\xi}{2}\frac{dp}{d\xi}=0,\\ \frac{d}{d\xi}\left(\lambda\frac{dT}{d\xi}\right)+\frac{k\rho}{\mu}\frac{dp}{d\xi}\frac{di}{d\xi}+\frac{a^2\xi}{2}\left(\rho m\frac{di}{d\xi}+C\frac{dT}{d\xi}\right)=0\end{gathered} \tag{7}$$

with the boundary conditions

$$p(\infty)=p_0,\quad p(0)=p_1,\quad T(\infty)=T_0,\quad T(0)=T_1. \tag{8}$$

Here, a^2 is the dimensional constant of diffusivity. Its form can be chosen in each specific case so that the equations of the problem will have the simplest form.

In particular, let the fluid saturating the pore space be a thermodynamically perfect gas, and let the porosity, permeability, viscosity, and thermal conductivity be independent of the pressure and temperature. Then

$$\rho=\rho_0 pT_0/(Tp_0),\quad i=C_pT \tag{9}$$

and Eqs. (7) take the form

$$\begin{gathered}\frac{k}{a^2\mu m}\frac{d}{d\xi}\left(\frac{p}{T}\frac{dp}{d\xi}\right)+\frac{\xi}{2T}\frac{dp}{d\xi}-\xi\frac{p}{2T^2}\frac{dT}{d\xi}=0,\\ \frac{\lambda}{Ca^2}\frac{d^2T}{d\xi^2}+\frac{k\rho_0T_0C_pp}{a^2\mu p_0CT}\frac{dp}{d\xi}\frac{dT}{d\xi}+\frac{\xi}{2}\left(\frac{\rho_0T_0C_ppm}{p_0TC}+1\right)\frac{dT}{d\xi}=0.\end{gathered} \tag{10}$$

We set

$$a^2=\frac{\lambda}{C},\quad p=p_0f,\quad T=T_0\Theta,\quad \chi=\frac{kp_0C}{\lambda\mu m},\quad \gamma=\frac{m\rho_0C_p}{C}.$$

We have the problem

$$\begin{gathered}\chi\frac{d}{d\xi}\left(\frac{f}{\Theta}\frac{df}{d\xi}\right)+\frac{1}{2}\frac{\xi}{\Theta}\frac{df}{d\xi}-\frac{\xi f}{2\Theta^2}\frac{d\Theta}{d\xi}=0,\\ \frac{d^2\Theta}{d\xi^2}+\chi\gamma\frac{f}{\Theta}\frac{df}{d\xi}\frac{d\Theta}{d\xi}+\frac{\xi}{2}\left(\gamma\frac{f}{\Theta}+1\right)\frac{d\Theta}{d\xi}=0,\\ \Theta(\infty)=f(\infty)=1,\quad \Theta(0)=\Theta_1=T_1/T_0,\quad f(0)=f_1=p_1/p_0.\end{gathered} \tag{11}$$

Let us examine the case when the perturbation is created by a thermal wave, $f_1=1$. If the intensity of the wave ($\varepsilon=|\Theta_1-\Theta_0|$) is not great, then the problem is solved by an expansion in the small parameter:

$$\begin{gathered}\Theta=1+\varepsilon\vartheta_0(\xi)+\varepsilon^2\vartheta_1(\xi)+\ \ldots,\\ f=1+\varepsilon\varphi_0(\xi)+\varepsilon^2\varphi_1(\xi)+\ \ldots\end{gathered} \tag{12}$$

Substitution into (11) gives

$$\begin{gathered}\frac{d^2\vartheta_0}{d\xi^2}+\frac{1}{2}\xi(1+\gamma)\frac{d\vartheta_0}{d\xi}=0,\\ \chi\frac{d^2\varphi_0}{d\xi^2}+\frac{1}{2}\xi\frac{d\varphi_0}{d\xi}=\frac{1}{2}\xi\frac{d\vartheta_0}{d\xi},\\ \frac{d^2\vartheta_1}{d\xi^2}+\frac{1}{2}(1+\gamma)\frac{d\vartheta_1}{d\xi}=-\chi\gamma\frac{d\varphi_0}{d\xi}\frac{d\vartheta_0}{d\xi},\end{gathered} \tag{13}$$

$$\vartheta_0(0)=1,\quad \vartheta_0(\infty)=\vartheta_1(0)=\vartheta_1(\infty)=\varphi_0(0)=\varphi_0(\infty)=0.$$

As a result, we have [2, 3]

$$\vartheta_0=1-\operatorname{erf}(\xi\sqrt{\gamma+1}/2),\quad \varphi_0=[\operatorname{erf}(\xi\sqrt{\gamma+1}/2)-\operatorname{erf}(\xi/2\sqrt{\chi})]/(\chi\gamma+\chi-1),$$
$$\vartheta_1=\int_\xi^\infty \exp\left(-\frac{1+\gamma}{4}\xi^2\right)\left[C_1-k_1\sqrt{\pi}\left(\operatorname{erf}\frac{\xi}{2\sqrt{\chi}}-\operatorname{erf}\frac{\sqrt{1+\gamma}}{2}\xi\right)d\xi,\right. \tag{14}$$

where

$$C_1=k_1\sqrt{1+\gamma}\int_0^\infty\left(\operatorname{erf}\frac{\xi}{2\sqrt{\chi}}-\operatorname{erf}\frac{\sqrt{\gamma+1}}{2}\xi\right)\exp\left(-\frac{\gamma+1}{4}\xi^2\right)d\xi;$$
$$k_1=\chi\gamma\sqrt{1+\gamma}/[\pi(\chi\gamma+\chi-1)]\quad \text{(see Fig. 1a)}.$$

The curves in Fig. 1 correspond to the parameters χ and γ, calculated with the values $\mu=17.2\cdot10^{-6}$, $C_p=10^3$ (for air at 20°C), $k=1.02\cdot10^{-15}$, $1.02\cdot10^{-16}$, $1.02\cdot10^{-17}$, $\lambda=1.75$, $m=0.25$, $C=2.72\cdot10^6$, $\rho_0=1$, $p_0=0.0981$, $\Delta T=300$°K, $T_0=293$°K.

As might be expected, the absolute values of the increment in intrapore pressure increase with a decrease in permeability and approach the values that would be obtained for an impermeable specimen (with the given temperature increment, the pressure maximum is 0.196 MPa).

The case in which heating occurs on the side of the boundary impermeable to gas is interesting in certain applications.

Formulation of the problem and the scheme of expansion in the small parameter are the same as before, with only the boundary condition for the dimensionless pressure changing to

$$f'(0)=\varphi_0'(0). \tag{15}$$

The corresponding solution, obtained by small-parameter expansion of the intensity of the thermal wave ε, has the form

$$\vartheta_0=1-\operatorname{erf}[\xi\sqrt{\gamma+1}/2],$$

$$\varphi_0=\frac{\sqrt{\gamma+1}}{\chi\gamma+\chi-1}\left(\sqrt{\chi}\operatorname{erf}\frac{\xi}{2\sqrt{\chi}}-\frac{1}{\sqrt{\gamma+1}}\operatorname{erfc}\frac{\xi\sqrt{\gamma+1}}{2}\right), \tag{16}$$
$$\vartheta_1=\int_0^\xi\exp\left(-\frac{\gamma+1}{4}\xi^2\right)\left[C_2+k_2\sqrt{\pi}\left(\frac{1}{\sqrt{\gamma+1}}\operatorname{erf}\frac{\xi\sqrt{\gamma+1}}{2}-\sqrt{\chi}\operatorname{erf}\frac{\xi}{2\sqrt{\chi}}\right)\right]d\xi,$$

where

$$C_2=-k_2\sqrt{\gamma+1}\int_0^\infty\exp\left(-\frac{\gamma+1}{4}\xi^2\right)\left(\frac{1}{\sqrt{\gamma+1}}\operatorname{erf}\frac{\xi\sqrt{\gamma+1}}{2}-\sqrt{\chi}\operatorname{erf}\frac{\xi}{2\sqrt{\chi}}\right)d\xi;$$
$$k_2=\chi\gamma(\gamma+1)/[\pi(\chi\gamma+\chi-1)]$$

(see Fig. 1b).

The pressure increment in the "sealed" specimen prove to be somewhat higher than in an open specimen for numerical values of the parameters coincident with those examined above, but this difference decreases at low permeabilities.

3. If the porous body is filled with an elastically compressible fluid instead of a gas, then for uniform motion we have the system of equations

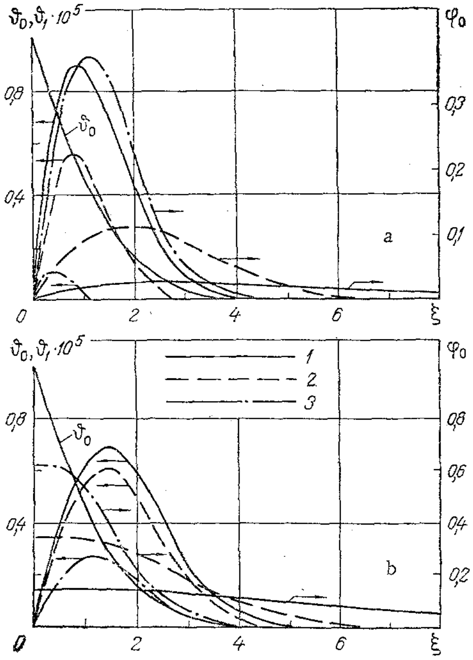

Fig. 1. The first two terms of the temperature expansion and the first term of the pressure expansion for the case of motion of a gas in a porous body: a) with a permeable surface; b) with an impermeable surface; 1) $k = 10^{-15}$; 2) 10^{-16}; 3) 10^{-17} m^2.

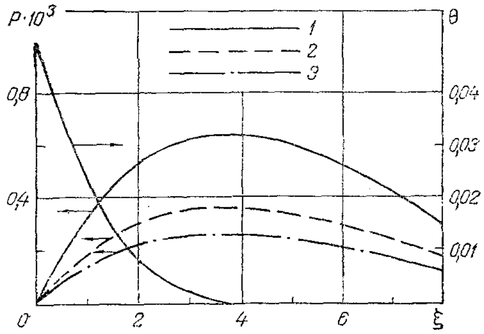

Fig. 2. Temperature and pressure perturbation for a fluid in a porous body with a permeable surface: 1) A = 100; 2) 173; 3) 250.

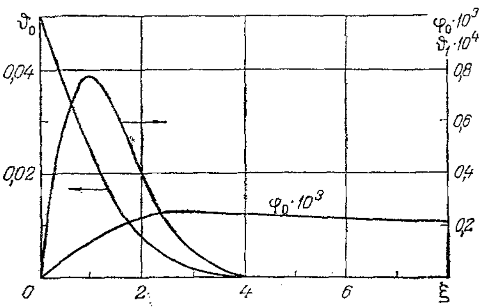

Fig. 3. The first two terms of the temperature expansion and the first term of the pressure expansion for the case of a fluid in a porous body with a permeable surface.

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial P}{\partial x}\left(\frac{\partial P}{\partial x}-\frac{\partial \Theta}{\partial x}\right)+\frac{1}{\varkappa}\left(\frac{\partial \Theta}{\partial x}-\frac{\partial P}{\partial t}\right)=0,$$

$$\frac{\chi}{m\varkappa}\frac{\partial^2\Theta}{\partial x^2}-\frac{1}{\varkappa}\left(\frac{C}{m\rho_0 C'}+1\right)\frac{\partial\Theta}{\partial t}+\frac{\partial\Theta}{\partial x}\frac{\partial P}{\partial x}=0. \quad (17)$$

Here

$$\rho=\rho_0[1+p/K-\alpha(T-T_0)];\quad P=p/K;\quad \Theta=\alpha(T-T_0);\quad \varkappa=kK/(m\mu);\quad \chi=\lambda/(C'\rho_0). \quad (18)$$

The problem of a thermal wave again permits a dynamically similar solution

$$P=P(\xi),\quad \Theta=\theta(\xi),\quad \xi=\frac{x}{a\sqrt{t}},\quad a^2=\frac{\chi C'\rho_0}{C+mC'\rho_0}\equiv\frac{\varkappa}{A}, \quad (19)$$

being a solution of the problem

$$\frac{d^2P}{d\xi^2}+\left(\frac{\xi}{2A}-\frac{d\Theta}{d\xi}+\frac{dP}{d\xi}\right)\frac{dP}{d\xi}-\frac{\xi}{2A}\frac{d\Theta}{d\xi}=0,$$

$$\frac{\chi}{m\varkappa}\frac{d^2\Theta}{d\xi^2}+\left(\frac{\chi}{2m\varkappa}\xi+\frac{dP}{d\xi}\right)\frac{d\Theta}{d\xi}=0,\quad A=\left(\frac{C}{C'm}+1\right)\frac{m\varkappa}{\chi}, \quad (20)$$

$$P(0)=P(\infty)=0,\quad \Theta(0)=\varepsilon,\quad \Theta(\infty)=0.$$

The results of calculation of the dynamically similar solution for the boundary-value problems $P(0) = P(\infty) = \Theta(\infty) = 0$, $\Theta(0) = 0.05$ and the parameter values $\mu = 0.1$, $k = 1.02 \cdot 10^{-15}$, $m = 0.25$, $\rho_0 = 1000$, $\lambda = 1.745$, $K = 1961$ MPa, $\alpha = 4.6 \cdot 10^{-4}$ (at 40-60°C), $C = 2.72 \cdot 10^6$, $C' = 4.17 \cdot 10^3$ are shown in Fig. 2.

It is obvious that, except for particularly "severe" regimes, the filtrative motions caused by the thermal shock will be weak, and they too can be analyzed by means of the small-parameter method. Assuming $\Theta(0) = \varepsilon \ll 1$, we have

$$\Theta=\vartheta_0+\varepsilon\vartheta_1+\ldots,\quad \vartheta_0(0)=\varepsilon,\quad \vartheta_0(\infty)=\vartheta_1(0)=\vartheta_1(\infty)=0, \quad (21)$$

$$P=P_0+\ldots,\quad P_0(0)=P_0(\infty)=0,$$

$$\frac{d^2\vartheta_0}{d\xi^2}+\frac{\xi}{2}\frac{d\vartheta_0}{d\xi}=0,\quad \frac{d^2P_0}{d\xi^2}+\frac{\xi}{2A}\frac{dP_0}{d\xi}=\frac{\xi}{2A}\frac{d\vartheta_0}{d\xi}, \quad (22)$$

$$\frac{\chi}{m\varkappa}\frac{d^2\vartheta_1}{d\xi^2}+\frac{\chi\xi}{2m\varkappa}\frac{d\vartheta_1}{d\xi}=-\frac{dP_0}{d\xi}\frac{d\vartheta_0}{d\xi}.$$

From which

$$\vartheta_0=\varepsilon[1-\operatorname{erf}(\xi/2)],\quad P_0=\frac{\varepsilon}{A-1}\left(\operatorname{erf}\frac{\xi}{2}-\operatorname{erf}\frac{\xi}{2\sqrt{A}}\right),$$

$$\vartheta_1=\int_\xi^\infty\left[C_3+\sqrt{\pi}\,k_3\left(\operatorname{erf}\frac{\xi}{2}-\operatorname{erf}\frac{\xi}{2\sqrt{A}}\right)\right]\exp\left(-\frac{\xi^2}{4}\right)d\xi, \quad (23)$$

where

$$C_3=k_3\int_0^\infty\left(\operatorname{erf}\frac{\xi}{2\sqrt{A}}-\operatorname{erf}\frac{\xi}{2}\right)\exp\left(-\frac{\xi^2}{4}\right)d\xi;\quad k_3=\frac{m\varkappa}{400\pi\chi(A-1)}$$

(see Fig. 3, $\varepsilon = 0.05$).

NOTATION

j, mass filtration velocity; p, fluid pressure, Pa; T, temperature; i, enthalpy; ρ, fluid density, kg/m^3; μ, fluid viscosity, $N \cdot sec/m^2$; m, porosity of the medium; k, permeability of the porous medium, m^2; λ, thermal conductivity of the fluid–porous-medium system, $W/m \cdot deg$; C, volumetric specific heat of the porous medium, $J/(m^3 \cdot deg)$; C_p, specific heat of the gas at constant pressure, $J/(kg \cdot deg)$; C', specific heat of the liquid, $J/(kg \cdot deg)$; K, compressive bulk modulus, N/m^2; α, coefficient of cubical expansion, deg^{-1}.

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STEADY-STATE PROBLEM OF LOCAL PORE COOLING

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The temperature field in a porous half-space with filtration of coolant from a source is examined.

Pore cooling has come into use in recent years in several sectors of modern industry to protect various structural elements from high heat fluxes. The high efficiency of this method of cooling is due to the developed surface with which the coolant is in contact during its motion through the porous medium. As a result of this, heat is absorbed, and the boundary layer at the leakage surface is transformed in such a way that heat transfer from the high-temperature gas flow to the wall being protected is reduced.

Together with the continuous supply of coolant through the wall [1], in our opinion coolant can also be supplied to certain local zones in some cases. This produces zonal pore cooling and creates the thermal regime required for the most heavily thermally stressed sections.

An important task in designing such systems is studying the temperature fields inside porous materials with allowance for the filtration processes occurring. To solve this problem, it is first necessary to construct the solution of the two-dimensional filtration problem and obtain the pressure distribution in the porous body. The heat-transfer equation can then be used with this data to find the temperature field.

Let us examine this problem using the example of coolant flow in a porous half-space (Fig. 1a [2]). We will assume that the cooling gas is moving in an undeformed, uniformly porous medium from a source of intensity 2M, located at point A, to the leakage surface. Constant pressure p_2 and temperature T_2 are maintained at the leakage surface, while the pressure and temperature at the source are p_1 and T_1, respectively. The thermophysical characteristics of the gas and the porous material are assumed to be constant, and equality is maintained between the temperatures of the body and the coolant at any point of the filtration region.

The gas flow in the porous medium obeys the resistance law

$$\frac{\alpha}{\mu(T)} \operatorname{grad} p = -\frac{f(V)}{V} \bar{V}. \quad (1)$$

The process of heat and mass transfer is described by the equations:

$$\lambda \Delta T - c\rho \bar{V} \operatorname{grad} T = 0, \quad (2)$$

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